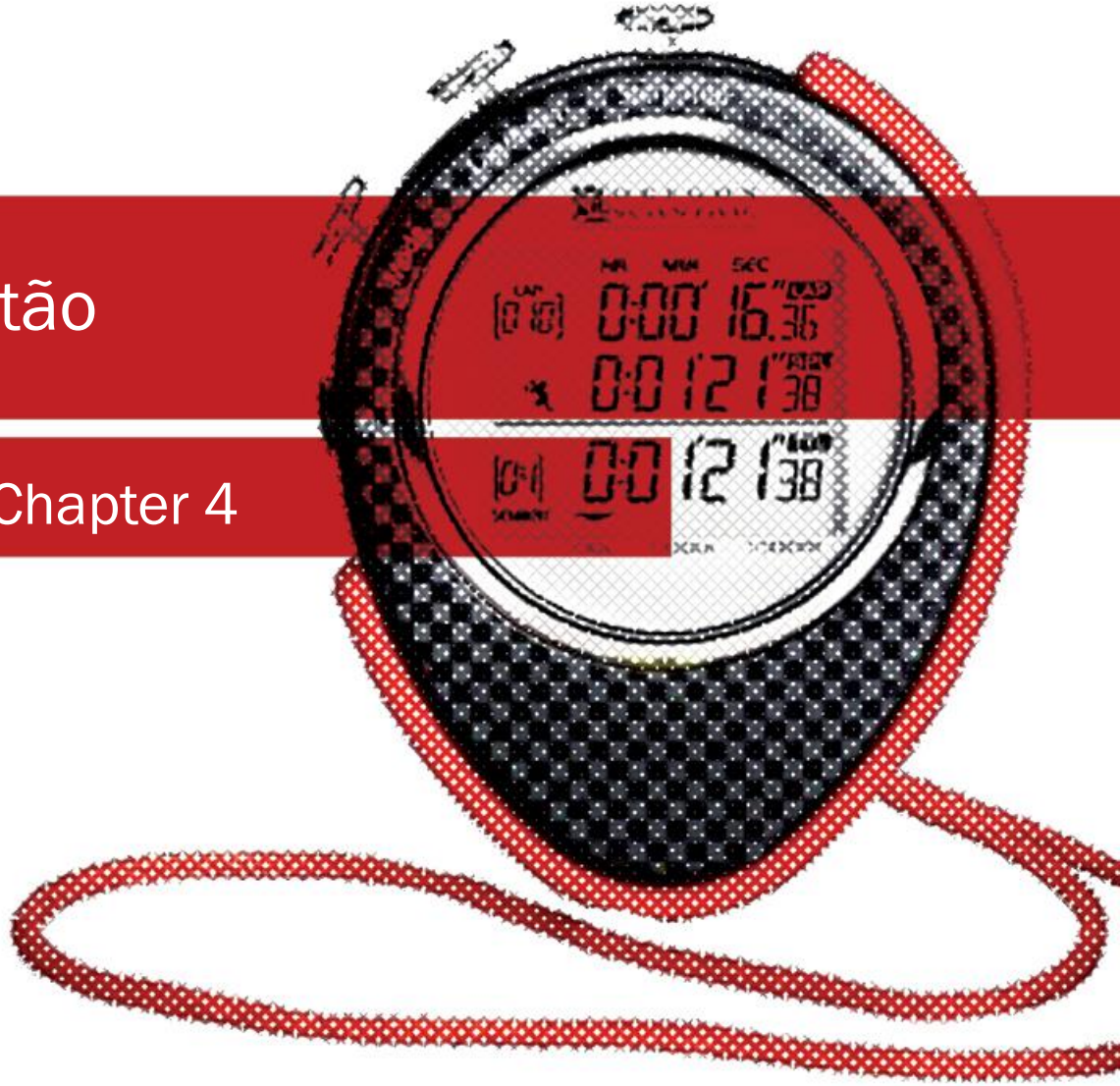
Licenciatura em Gestão

Operational Research Chapter 4

2014-2015



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4. The Transportation and the Assignment Problems

4.1 Introduction

4.2 Transportation Problem

4.3 Assignment Problem

The Transportation Problem

Transportation Problem (**TP**) — determine the quantities of an homogeneous commodity to be shipped from a set of **distribution centres** — **the origins (sources)** — to a set of **receiving centres** — **the destinations** — such that the total cost is minimised.

Applications:

- Products transportation;
- Production planning;
- Scheduling human resources;
- ...

The Transportation Problem

Data: m = number of origins; s_i = **supply** at origin i;
n = number of destinations; d_j = **demand** at destination j;
c_{ij} = cost per unit distributed from i to j

Assumption: the TP is **balanced**, that is, the total supply and total demand are equal.

x_{ij} – amount to ship from origin i to destination j.

Z – total cost of the transportation plan

LP formulation:

$$\text{Minimize } Z=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = s_i \quad (i=1,\ldots,m) \\ \sum_{i=1}^{m} x_{ij} = d_j \quad (j=1,\ldots,n) \\ x_{ij} \geq 0 \ (i=1,\ldots,m;\ j=1,\ldots,n) \end{cases}$$

TP Prototype Example

	A	B	C	D	E	F	G	H	I	J
1	P&T Co. Distribution Problem									
2										
3					shipping cost (m.u./per truck load)					
4						dest ination				
5				W1	W2	W3	W4	supply		
6			C1	464	513	654	867	75		
7		origin	C2	352	416	690	791	125		
8			C3	995	682	388	685	100		
9			demand	80	65	70	85			300
10										300
11										
12					solution (quant. to be shipped)					
13						dest ination				
14				W1	W2	W3	W4	total		supply
15			C1	0	0	0	0	0	=	75
16		origin	C2	0	0	0	0	0	=	125
17			C3	0	0	0	0	0	=	100
18			total	0	0	0	0	0	=	COST
19				=	=	=	=			
20			demand	80	65	70	85			
21										

J9=SUM(H6:H8)

J10=SUM(D9:G9)

H15=SUM(D15:G15)

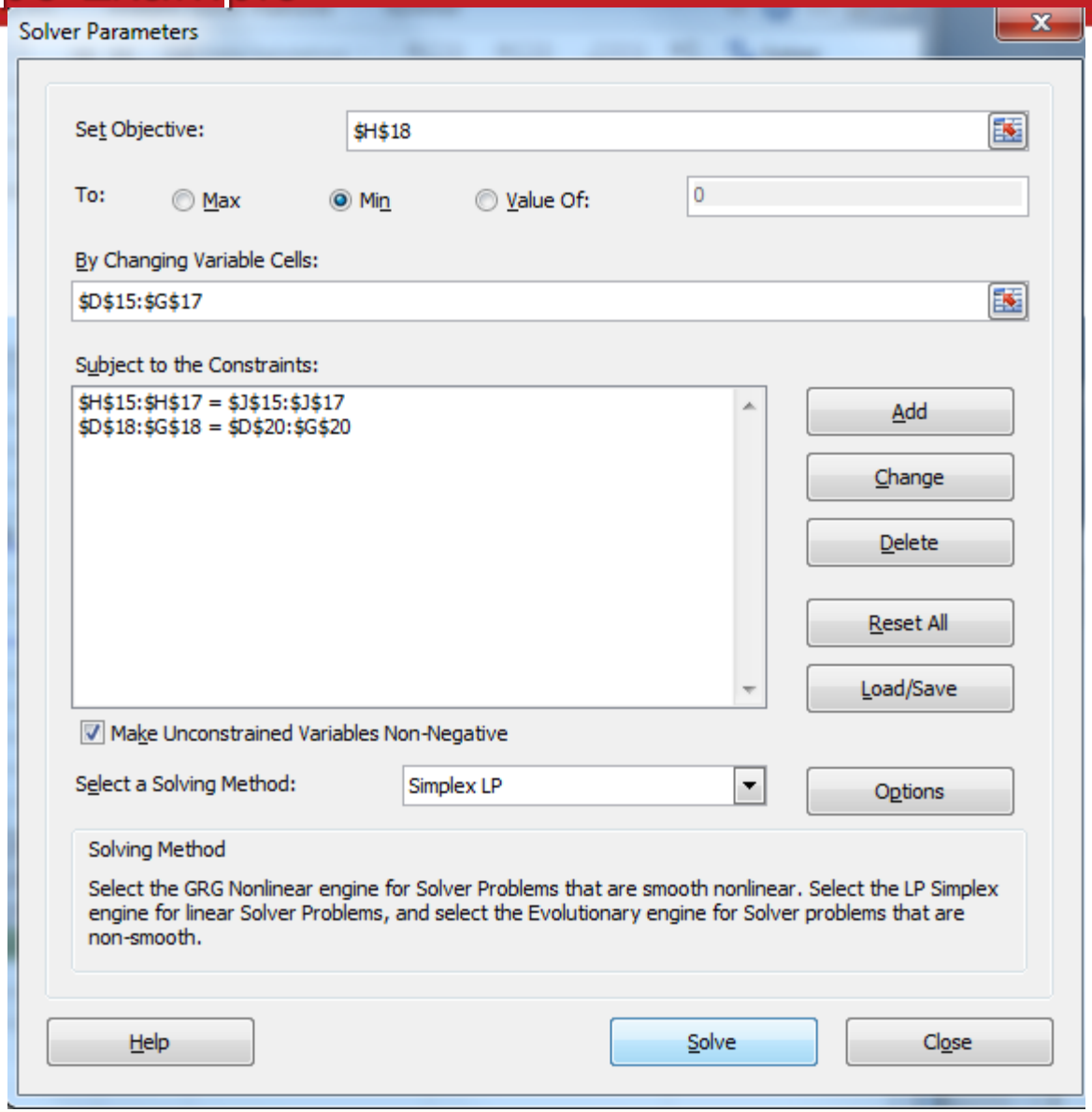
H16=SUM(D16:G16)

H17=SUM(D17:G17)

H18=SUMPRODUCT(D6:G8;D15:G17)

D18=SUM(D15:D17) E18=SUM(E15:E17) F18=SUM(F15:F17) G18=SUM(G15:G17)

TP Prototype Example



TP Prototype Example reports

Microsoft Excel 8.0e Answer Report
Worksheet: [PT_prot_P&T.xls]Sheet1
Report Created: 29-04-2003 20:06:05

Target Cell (Min)

Cell	Name	Original Value	Final Value
$H$18	total total	0	152535

Adjustable Cells

Cell	Name	Original Value	Final Value
$D$15	F1 A1	0	0
$E$15	F1 A2	0	20
$F$15	F1 A3	0	0
$G$15	F1 A4	0	55
$D$16	F2 A1	0	80
$E$16	F2 A2	0	45
$F$16	F2 A3	0	0
$G$16	F2 A4	0	0
$D$17	F3 A1	0	0
$E$17	F3 A2	0	0
$F$17	F3 A3	0	70
$G$17	F3 A4	0	30

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$15	F1 total	75	$H$15=$J$15	Binding	0
$H$16	F2 total	125	$H$16=$J$16	Binding	0
$H$17	F3 total	100	$H$17=$J$17	Not Binding	0
$D$18	total A1	80	$D$18=$D$20	Not Binding	0
$E$18	total A2	65	$E$18=$E$20	Not Binding	0
$F$18	total A3	70	$F$18=$F$20	Not Binding	0
$G$18	total A4	85	$G$18=$G$20	Not Binding	0

Microsoft Excel 8.0e Sensitivity Report
Worksheet: [PT_prot_P&T.xls]Sheet1
Report Created: 29-04-2003 20:06:05

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$15	F1 A1	0	15	464	1E+30	15
$E$15	F1 A2	20	0	513	15	21
$F$15	F1 A3	0	84	654	1E+30	84
$G$15	F1 A4	55	0	867	21	351
$D$16	F2 A1	80	0	352	15	1E+30
$E$16	F2 A2	45	0	416	21	15
$F$16	F2 A3	0	217	690	1E+30	217
$G$16	F2 A4	0	21	791	1E+30	21
$D$17	F3 A1	0	728	995	1E+30	728
$E$17	F3 A2	0	351	682	1E+30	351
$F$17	F3 A3	70	0	388	84	1E+30
$G$17	F3 A4	30	0	685	351	84

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$H$15	F1 total	75	570	75	0	55
$H$16	F2 total	125	473	125	0	45
$H$17	F3 total	100	388	100	0	70
$D$18	total A1	80	-121	80	45	0
$E$18	total A2	65	-57	65	55	0
$F$18	total A3	70	0	70	0	1E+30
$G$18	total A4	85	297	85	70	0

The Transportation Problem

Properties of the TP

P1: The TP has at least one feasible solution.

Corollary: The TP has optimal solution.

P2: A TP where **supplies** and **demands** have integer values has at least one optimal solution with all variables integer valued.

The Transportation Problem - Variants

Problems that have the same structure of parameters but:

(V1) total supply > total demand: origin constraints "≤".

Opt. Sol. : Part of the supply is not transported

(V2) total supply < total demand: destination constraints "≤".

Opt. Sol. : Part of the demand is not satisfied

(V3) Destination requiring demand between a minimum and a maximum value:

2 constraints at the destination: "≤ maximum demand" and "≥ minimum demand".

(V4) Origin producing supply between a minimum and a maximum value: ≈ (V3)

(V5) Infeasible link: corresponding variable is set to zero.

(V6) Maximization problem: in solver/excel choose OF: Max. ⊙

The Assignment Problem

Assignment Problem (AP)

Assign ***n*** people to ***n*** tasks – each person to a task, each task to a person – minimising the total assignment cost.

Aplications:

- Assign people to tasks;
- Production planning (operations to machines; products to plants)
- ….

The Assignment Problem

Data: n = number of persons and tasks

c_{ij} = cost associated with assignee i performing task j (i,j=1,…,n)

$$x_{ij} = \begin{cases} 1 & \text{if person } i \text{ is assigned to job } j \\ 0 & \text{otherwise} \end{cases}$$

Z – total cost of the assignment plan

LP Formulation:

$$\text{Minimize } Z=\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = 1 \quad (i=1,\ldots,n) \\ \sum_{i=1}^{n} x_{ij} = 1 \quad (j=1,\ldots,n) \\ x_{ij} \geq 0 \quad (i,j=1,\ldots,n) \\ x_{ij} \text{ binary } (i,j=1,\ldots,n) \end{cases}$$

The Assignment Problem - Variants

(V1) number of people > number of tasks: people constraints "≤".

Opt.Sol.: some people is not assigned

(V2) number of people < number of tasks : task constraints "≤".

OS: some tasks are not performed

(V3) some task can be assigned to more than one person

respective constraint " ≥1 "

(V4) some person can perform more than one task ≈ (V3)

(V5) Infeasible links: person *i* cannot be assigned to task *j* then x_{ij}=0.

(V6) Maximization problem: in solver/excel choose OF TO: Max. ⊙